Effective field theory approach to light propagation in an external magnetic field

Xue-Peng Hu, Yi Liao[a]

Department of Physics, Nankai University, Tianjin 300071, P.R. China

Abstract. The recent PVLAS experiment observed rotation of polarization and ellipticity when a linearly polarized laser beam passes through a transverse magnetic field. The phenomenon cannot be explained in conventional QED. We attempt to accommodate the result by employing an effective theory for the electromagnetic field alone. No new particles with a mass of order the laser frequency or below are assumed. To quartic terms in the field strength, a parity-violating term appears besides the two ordinary terms. The rotation of polarization and ellipticity are computed for parity-asymmetric and -symmetric experimental set-ups. While rotation occurs in an ideal asymmetric case and has the same magnitude as ellipticity, it disappears in a symmetric set-up like PVLAS. This would mean that we have to appeal to some low-mass new particles with nontrivial interactions with photons to understand the PVLAS result.

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1 Introduction

As a consequence of quantum effects, the neutral photons can interact and lead to nonlinear phenomena in vacuum in the presence of an electromagnetic field [1–5]. Recently the PVLAS experiment has observed the rotation of polarization [6] and ellipticity [7] on passing a linearly polarized laser light in vacuum through a transverse magnetic field. Surprisingly, the observed ellipticity is larger by a factor of 10^4 than predicted in the conventional QED, and a rotation of similar magnitude has also been observed, which, however, is not expected at the leading order of the weak field expansion in QED [3, 4, 8, 9].

There have been some theoretical attempts to understand the PVLAS results. The best motivated one might be the interpretation in terms of axion-like particles [10–13] as the axion also offers a potential solution to the strong CP problem and serves as one of the leading candidates for dark matter. Nevertheless, its mass and coupling to two photons are strongly constrained by astrophysical observations; for a recent review, see, e.g., [14]. For instance, the parameters preferred by the PVLAS results would be in the region excluded by the negative results of CAST, which searches for axion-like particles coming from the Sun [15]. A straightforward interpretation thus seems not to be possible. Solutions to this dilemma have been suggested that invoke novel suppression mechanisms for axion emission in a stellar medium [16–18] or its propagation from the stars to the Earth [19], para-photons to induce different

effective charges to particles in vacuum and in stellar plasmas [20], or one introduces a more specific structure in the photon–(pseudo-) scalar sector [21]. Or more directly, low-mass milli-charged particles can also contribute via real production or virtual processes to the quantities observed by PVLAS without conflicting with astrophysical observations [22, 23]. A possible means to test the idea of axion-like particles might be photon regeneration experiments [10, 11, 24–26]. For some recent discussions on this, see e.g. [27–29] for a partial list of references.

Instead of introducing more hypothetical low-mass particles and endowing them with more complicated properties, we consider it natural to ask whether it is possible to understand the PVLAS results within the realm of photon interactions. Our idea is to work with an effective field theory for photons alone at the energy scale of order eV as specified by the laser light frequency and investigate the feasibility to incorporate the results on the rotation of polarization and ellipticity. The effective theory generally contains some free parameters that are ultimately determined by a certain fundamental theory defined at a higher mass scale. If this is feasible, the next step would be to single out the fundamental theory that can reproduce the parameters favored by experiments. If it is not, it would be inevitable to introduce particles of mass lower than an eV with some exotic properties if the experiments are to be explained within particle physics.

The paper is organized as follows. In the next section we write down the most general Lagrangian for electromagnetic fields to the lowest non-trivial order that is consistent with gauge and Lorentz symmetries. It turns out to con-

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[a] e-mail: liaoy@nankai.edu.cn

tain a parity-violating term. The equation of motion for the laser light traversing a transverse magnetic field is then set up, and its propagation eigenmodes are found. The eigenmodes are employed in Sect. 3 to study the propagation of the linearly polarized light in an applied magnetic field that is transverse to the propagation direction. The dichroism (rotation of the polarization plane) and birefringence (ellipticity) are computed for two types of experimental set-ups, one for a Gedanken experiment with light propagating forward in one direction and the other for the PVLAS with light traveling forth and back in a Fabry–Pérot cavity. Our results are summarized and conclusions are made in the last section.

2 Equation of motion and propagation eigenmodes

We assume that there are no other particles than the photon with mass of order eV (the laser light frequency) or lower. The gauge and Lorentz invariant effective Lagrangian that describes low energy photon interactions is an expansion in the field tensor $F_{\mu\nu}$ and partial derivatives. For terms with a given number of $F_{\mu\nu}$, those with additional partial derivatives are suppressed relative to those without, either by the low frequency of the laser light or the slow variation of the external electromagnetic field compared to the energy scale of some underlying theory. We can thus ignore all terms with additional derivatives.

Working to quartic terms in the electromagnetic field, we have in unrationalized Gaussian units with $\hbar = c = 1$

$$4\pi\mathcal{L} = -\mathcal{F} + 2\kappa_1\mathcal{F}^2 + 2\kappa_2\mathcal{G}^2 + 4\kappa_3\mathcal{F}\mathcal{G}\,, \qquad (1)$$

where $\mathcal{F} = \frac{1}{4}F^{\mu\nu}F_{\mu\nu}$, $\mathcal{G} = \frac{1}{4}\tilde{F}^{\mu\nu}F_{\mu\nu}$ and $\tilde{F}^{\mu\nu} = \frac{1}{2}\epsilon^{\mu\nu\alpha\beta}F_{\alpha\beta}$, with $\epsilon^{0123} = +1$. The identification with the classical field strengths in vacuum is $F^{0i} = -E^i$, $F^{ij} = -\epsilon^{ijk}B^k$ with $\epsilon^{123} = +1$; then, $\tilde{F}^{0i} = -B^i$, $\tilde{F}^{ij} = \epsilon^{ijk}E^k$ and $\mathcal{F} = \frac{1}{2}(\mathbf{B}^2 - \mathbf{E}^2)$, $\mathcal{G} = \mathbf{E} \cdot \mathbf{B}$. With four factors of F and $\tilde{F}$, there are two ways to contract Lorentz indices; one contains two chains of contraction as shown, and the other one has only one chain like $F_{\mu\rho}F^{\rho\sigma}F_{\sigma\alpha}F^{\alpha\mu}$. But the latter structures are not independent because they can be reduced to combinations of the former:

$$\begin{aligned} F_{\mu\rho}F^{\rho\sigma}F_{\sigma\alpha}F^{\alpha\mu} &= 8\mathcal{F}^2 + 4\mathcal{G}^2 \\ \tilde{F}_{\mu\rho}F^{\rho\sigma}F_{\sigma\alpha}F^{\alpha\mu} &= -4\mathcal{F}\mathcal{G} \\ \tilde{F}_{\mu\rho}\tilde{F}^{\rho\sigma}F_{\sigma\alpha}F^{\alpha\mu} &= 4\mathcal{G}^2\,. \end{aligned} \qquad (2)$$

It is not necessary to consider structures with more tildes as they are related to the above by $\mathcal{F} \to -\mathcal{F}$, $\mathcal{G} \to -\mathcal{G}$ and thus do not yield independent ones. The above quartic terms are the most general ones consistent with gauge and Lorentz symmetry.

It is not possible to form a term with an odd number of F and $\tilde{F}$ without additional derivatives. Such a term would violate the charge conjugation symmetry (C). To construct such a term we would need at least one chain of contraction involving an odd number of F and $\tilde{F}$, which

however vanishes identically due to antisymmetry of F and $\tilde{F}$. The quartic coefficients κ_i have the dimensions of mass^{-4} and should be computed from some underlying theory. In particular, the κ_3 term violates the parity P (and thus CP) and should presumably be small compared to κ_1 and κ_2. We would like to emphasize that our effective Lagrangian describes physics at the energy scale of order eV and is not suitable as virtual insertions, for instance, into the electromagnetic moments of the electron and muon, which involve energy scales of at least the lepton masses. Finally, the leading order terms in the Heisenberg–Euler Lagrangian for conventional QED[1] is reproduced by

$\kappa_1 = \dfrac{\alpha^2}{45\pi m_e^4}$, $\kappa_2 = \frac{7}{4}\dfrac{\alpha^2}{45\pi m_e^4}$ and $\kappa_3 = 0$, where m_e is the electron mass.

The equation of motion is

$$\begin{aligned} &\partial_\alpha F^{\alpha\beta} \\ &= 4\partial_\alpha\left[\kappa_1\mathcal{F}F^{\alpha\beta} + \kappa_2\mathcal{G}\tilde{F}^{\alpha\beta} + \kappa_3\left(\mathcal{G}F^{\alpha\beta} + \mathcal{F}\tilde{F}^{\alpha\beta}\right)\right]. \end{aligned} \qquad (3)$$

Now decompose the field into a quantum part denoted by lower-case letters plus a classical part denoted by capital ones. They will be identified with the laser light and the applied magnetic field, respectively. The equation of motion contains terms that are zeroth, first and second order in the quantum field. The zeroth order terms specify the applied external field and are of no interest here. The second order terms are too small compared to the first order ones. What remains is thus the linearized equation of motion for the quantum field:

$$\begin{aligned} \partial_\alpha f^{\alpha\beta} = \partial_\alpha\Big\{ &\kappa_1\left[2(fF)F^{\alpha\beta} + (FF)f^{\alpha\beta}\right] \\ &+ \kappa_2\left[2(\tilde{F}f)\tilde{F}^{\alpha\beta} + (\tilde{F}F)\tilde{f}^{\alpha\beta}\right] \\ &+ \kappa_3\big[2(\tilde{F}f)F^{\alpha\beta} + (\tilde{F}F)f^{\alpha\beta} + 2(Ff)\tilde{F}^{\alpha\beta} \\ &+ (FF)\tilde{f}^{\alpha\beta}\big]\Big\}, \end{aligned} \qquad (4)$$

where $(XY) \equiv X_{\mu\nu}Y^{\mu\nu}$ with $X, Y = f, F, \tilde{f}, \tilde{F}$.

Now we specialize to the case of a static, spatially uniform external magnetic field $\mathbf{B}$. Although PVLAS applies a rotating constant magnetic field, it has been shown [8, 9] that it can be treated as static due to its low frequency. We choose the gauge $a^\mu = (0, \mathbf{a})$ with $\nabla \cdot \mathbf{a} = 0$ for the light potential 4-vector. We assume that the plane wave light propagates in the z axis and $\mathbf{B}$ points in the x axis. Then, the only nontrivial equation is for the components in the (x, y) plane, a_1 and a_2, which depend on (t, z) as a plane wave. After some algebra, it simplifies to

$$\begin{aligned} (\partial_t^2 - \partial_z^2)a_i = &\frac{1}{2}\chi_1(\partial_t^2 - \partial_z^2)a_i - \chi_1\delta_{i2}\partial_z^2 a_2 \\ &- \chi_2\delta_{i1}\partial_t^2 a_1 + \chi_3\partial_t\partial_z\left(\delta_{i2}a_1 + \delta_{i1}a_2\right), \end{aligned} \qquad (5)$$

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[1] For a modern introduction to the Heisenberg–Euler Lagrangian in the conventional QED, see, e.g., [30].

where $\chi_i \equiv 4\kappa_i \mathbf{B}^2$, or in components,

$$(\partial_t^2 - \partial_z^2) \begin{pmatrix} a_1 \\ a_2 \end{pmatrix} =$$

$$\begin{pmatrix} \frac{1}{2}\chi_1(\partial_t^2 - \partial_z^2) - \chi_2\partial_t^2 & \chi_3\partial_t\partial_z \\ \chi_3\partial_t\partial_z & \frac{1}{2}\chi_1(\partial_t^2 - \partial_z^2) - \chi_1\partial_z^2 \end{pmatrix} \begin{pmatrix} a_1 \\ a_2 \end{pmatrix}. \tag{6}$$

For the laser propagating in the $+z$ direction, we can replace $a_i \to a_i \mathrm{e}^{\mathrm{i}(kz-\omega t)}$. For $|\chi_{1,2,3}| \ll 1$, which is the case here, the modification by $\mathbf{B}$ is small; then it is safe to ignore the $(k^2 - \omega^2)$ terms and identify ω with k on the right hand side of the equation. The error of this approximation is $O(\kappa_i^2 \mathbf{B}^4)$ which goes beyond the precision of our working Lagrangian (1). The eigenvalue equation simplifies to

$$\begin{pmatrix} (\omega^2 - k^2) + k^2\chi_2 & k^2\chi_3 \\ k^2\chi_3 & (\omega^2 - k^2) + k^2\chi_1 \end{pmatrix} \begin{pmatrix} a_1 \\ a_2 \end{pmatrix} = 0. \tag{7}$$

The dispersion relations are determined by the vanishing determinant of the coefficient matrix,

$$\omega_\pm = k\sqrt{1 - \bar{\chi} \pm \sqrt{(\Delta\chi)^2 + \chi_3^2}}, \tag{8}$$

where

$$\bar{\chi} = \frac{1}{2}(\chi_1 + \chi_2), \ \Delta\chi = \frac{1}{2}(\chi_1 - \chi_2), \tag{9}$$

and the eigenvectors are accordingly

$$\begin{pmatrix} a_1 \\ a_2 \end{pmatrix}_\pm = \begin{pmatrix} \sin\delta_\pm \\ \cos\delta_\pm \end{pmatrix}, \tag{10}$$

where

$$\sin\delta_\pm = \frac{\chi_3}{\sqrt{\left[\Delta\chi \mp \sqrt{(\Delta\chi)^2 + \chi_3^2}\right]^2 + \chi_3^2}},$$

$$\cos\delta_\pm = \frac{\Delta\chi \mp \sqrt{(\Delta\chi)^2 + \chi_3^2}}{\sqrt{\left[\Delta\chi \mp \sqrt{(\Delta\chi)^2 + \chi_3^2}\right]^2 + \chi_3^2}}. \tag{11}$$

Two comments are in order. First, in contrast to the QED case, the propagation eigenmodes are generally not those parallel and perpendicular respectively to the applied $\mathbf{B}$ field. Second, for the laser propagating in the $-z$ direction, the solutions are obtained by flipping the sign of χ_3. This is because the χ_3 term in the original equation of motion is linear in the z-derivative. These features arise from the parity-violating term in our Lagrangian.

3 Dichroism and birefringence

Having found the propagation eigenmodes of light in a transverse constant magnetic field, we can now follow its evolution from a given initial state. To see the effects of the κ_3 term clearly, we consider two experimental set-ups, one for an ideal case and the other for the PVLAS.

3.1 One-direction propagation

Assuming that the laser light is linearly polarized with an angle θ to $\mathbf{B} = |\mathbf{B}|\hat{x}$ before entering the field and propagates in the $+z$ direction, the potential vector for the light is

$$\mathbf{a}(0, z) = a_0 \mathrm{e}^{\mathrm{i}kz} \begin{pmatrix} \cos\theta \\ \sin\theta \end{pmatrix}. \tag{12}$$

Decomposing the light traversing the field as a sum of the propagation eigenmodes,

$$\mathbf{a}(t, z) = c_+ \mathrm{e}^{\mathrm{i}(kz-\omega_+ t)} \begin{pmatrix} \sin\delta_+ \\ \cos\delta_+ \end{pmatrix}$$

$$+ c_- \mathrm{e}^{\mathrm{i}(kz-\omega_- t)} \begin{pmatrix} \sin\delta_- \\ \cos\delta_- \end{pmatrix}, \tag{13}$$

the initial condition determines

$$c_\pm = \pm a_0 \frac{\cos(\theta + \delta_\mp)}{\sin(\delta_+ - \delta_-)}. \tag{14}$$

After some algebra, the light exiting the $\mathbf{B}$ field at $(t, z) = (T, L)$ is found to be

$$\mathbf{a}(T, L) = a_0 \mathrm{e}^{\mathrm{i}(kL - \bar{\omega}T)} \begin{pmatrix} r_1 \mathrm{e}^{\mathrm{i}\phi_1} \\ r_2 \mathrm{e}^{\mathrm{i}\phi_2} \end{pmatrix}, \tag{15}$$

with

$$r_1 \mathrm{e}^{\mathrm{i}\phi_1} = \cos(T\Delta\omega)\cos\theta - \mathrm{i}\sin(T\Delta\omega)\cos(\theta + \delta),$$
$$r_2 \mathrm{e}^{\mathrm{i}\phi_2} = \cos(T\Delta\omega)\sin\theta + \mathrm{i}\sin(T\Delta\omega)\sin(\theta + \delta), \tag{16}$$

where the following quantities are defined:

$$\bar{\omega} = \frac{1}{2}(\omega_+ + \omega_-) \approx k\left(1 - \frac{1}{2}\bar{\chi}\right),$$
$$\Delta\omega = \frac{1}{2}(\omega_+ - \omega_-) \approx \frac{1}{2}k\sqrt{(\Delta\chi)^2 + \chi_3^2},$$
$$\cos\delta = \frac{\Delta\chi}{\sqrt{(\Delta\chi)^2 + \chi_3^2}},$$
$$\sin\delta = \frac{\chi_3}{\sqrt{(\Delta\chi)^2 + \chi_3^2}}. \tag{17}$$

For the laser entering the $\mathbf{B}$ field at $(t, z) = (0, 0)$, we can identify $T \approx L$.

The rotation of the polarization is measured by $(\alpha - \theta)$ with

$$\tan\alpha = \frac{r_2}{r_1}$$
$$= \sqrt{\frac{\cos^2(T\Delta\omega)\sin^2\theta + \sin^2(T\Delta\omega)\sin^2(\theta + \delta)}{\cos^2(T\Delta\omega)\cos^2\theta + \sin^2(T\Delta\omega)\cos^2(\theta + \delta)}}, \tag{18}$$

while the ellipticity is measured by $\tan\beta$ with

$$\sin 2\beta = \sin 2\alpha \sin(\phi_2 - \phi_1). \tag{19}$$

For $T\Delta\omega \ll 1$, we generally have $\tan\alpha = \tan\theta + O((T\Delta\omega)^2)$; i.e., there is no rotation to $O(T\Delta\omega)$. Using $\phi_2 - \phi_1 \approx T\Delta\omega \dfrac{\sin(2\theta+\delta)}{\sin\theta\cos\theta}$, we obtain

$$\beta \approx T\Delta\omega \sin(2\theta+\delta) \approx \frac{1}{2}\bar\omega T\left(\Delta\chi \sin 2\theta + \chi_3 \cos 2\theta\right). \tag{20}$$

Note that the parity-conserving and parity-violating contributions depend differently on the angle between the polarization of light and the **B** field.

Interesting cases occur for $\sin 2\theta = 0$, where the above manipulations do not apply. These are the ideal cases when the angle θ can be measured to a precision better than the small quantity $T\Delta\omega$. At $\theta = 0$, we have $\frac{r_2}{r_1} \approx \frac{1}{2}\bar\omega T|\chi_3|$, $\phi_2 - \phi_1 \approx \frac{\pi}{2}\mathrm{sign}(\chi_3) + \frac{1}{2}\bar\omega T\Delta\chi$ so that

$$\alpha \approx \frac{1}{2}\bar\omega T|\chi_3|, \quad \beta \approx \alpha\,\mathrm{sign}(\chi_3) \approx \frac{1}{2}\bar\omega T\chi_3. \tag{21}$$

At $\theta = \frac{1}{2}\pi$, we find similarly

$$\alpha \approx \frac{\pi}{2} - \frac{1}{2}\bar\omega T|\chi_3|, \quad \beta \approx -\frac{1}{2}\bar\omega T\chi_3. \tag{22}$$

In these cases, the rotation and ellipticity have the same magnitude but differ in sign when $\chi_3 < 0$ and originate completely from the parity-violating term in the Lagrangian.

3.2 Round-trip propagation

Since the optical effects discussed here are very small, it is not practical in the laboratory experiments to carry out a one-way propagation of the laser light. The PVLAS experiment employs a high-finesse Fabry–Pérot cavity that allows the light to propagate forth and back about $N \sim 4.4 \times 10^4$ times before exiting the magnetic field and thus prolongs the optical path significantly. Our previous discussion assumes a linearly polarized initial state without ellipticity and does not necessarily apply to the round-trip set-up. This is because the rotation and ellipticity accumulated from a previous trip serves as an initial state for the subsequent one and may affect their further accumulation. Now we treat this circumstance carefully.

Consider the laser light propagating in the $+z$ direction with the initial condition

$$\mathbf{a}(0, z) = a_0 e^{ikz} \begin{pmatrix} e^{-i\phi}\cos\theta \\ e^{+i\phi}\sin\theta \end{pmatrix}. \tag{23}$$

On exiting the **B** field, it becomes

$$\mathbf{a}(T, L) = a_0 e^{i(kL - \bar\omega T)} \begin{pmatrix} r_1 e^{i\phi_1} \\ r_2 e^{i\phi_2} \end{pmatrix}, \tag{24}$$

where

$$r_1 e^{i\phi_1} = [Cc_\phi c_\theta - Ss_\phi c_{\theta-\delta}] - i[Sc_\phi c_{\theta+\delta} + Cs_\phi c_\theta],$$
$$r_2 e^{i\phi_2} = [Cc_\phi s_\theta - Ss_\phi s_{\theta-\delta}] + i[Sc_\phi s_{\theta+\delta} + Cs_\phi s_\theta] \tag{25}$$

and $C = \cos(T\Delta\omega)$, $S = \sin(T\Delta\omega)$, $c_\phi = \cos\phi$, $s_\phi = \sin\phi$, etc. Denoting $t_\theta = \tan\theta$, $\rho = c_\delta \tan(T\Delta\omega)$ and $\sigma = s_\delta \tan(T\Delta\omega)$, we have

$$\frac{r_2 e^{i\phi_2}}{r_1 e^{i\phi_1}} = \frac{(1+i\rho)t_\theta e^{i2\phi} + i\sigma}{(1-i\rho) + i\sigma t_\theta e^{i2\phi}}. \tag{26}$$

For each single trip in the Fabry–Pérot cavity, $|\rho|$ and $|\sigma|$ are tiny. Assuming $t_\theta \leq 1$, we can expand the above in ρ and σ (shifting to $t_\theta^{-1}e^{-i2\phi}$ if $t_\theta > 1$):

$$\frac{r_2 e^{i\phi_2}}{r_1 e^{i\phi_1}} \approx t_\theta e^{i2\phi}\left[1 + i2\rho + i2\sigma c_{2\theta}\frac{c_{2\theta}}{s_{2\theta}} + 2\sigma s_{2\phi}\frac{1}{s_{2\theta}}\right], \tag{27}$$

which should be identified with

$$t_{\theta+\delta\theta}e^{i2(\phi+\delta\phi)} \approx t_\theta e^{i2\phi}\left[1 + i2\delta\phi + \frac{2\delta\theta}{s_{2\theta}}\right]. \tag{28}$$

This gives the increments from a single trip in the $+z$ direction:

$$\delta\phi \approx \rho + \sigma c_{2\phi}t_{2\theta}^{-1}, \quad \delta\theta \approx \sigma s_{2\phi}. \tag{29}$$

A trip in the $-z$ direction from the same initial condition will yield the increments that are obtained by flipping the sign of $\sigma \propto s_\delta$:

$$\delta\phi \approx \rho - \sigma c_{2\phi}t_{2\theta}^{-1}, \quad \delta\theta \approx -\sigma s_{2\phi}. \tag{30}$$

For a round-trip, the values of ϕ and θ in the trigonometric functions multiplying σ of the return-trip increments can be identified with the initial ones, because the pre-factors ρ and σ are already at $O(\chi_j)$. The errors caused by this are of $O(|\mathbf{B}|^4)$ and are neglected here from the very start. Then a round-trip cancels the σ terms and doubles the ρ term in $\delta\theta$ and $\delta\phi$. There is thus no rotation from a round-trip. The laser light in PVLAS travels in the cavity an odd number N of times before exit. Since N is very large, the contribution from the unpaired single trip can be neglected compared to the $\frac{1}{2}(N-1) \approx \frac{1}{2}N$ times of the round-trip. Thus, the final ellipticity is

$$\beta_N \approx \frac{1}{2}N\bar\omega L\Delta\chi \sin 2\theta, \tag{31}$$

where L is the length of a single pass in the **B** field. Note that the ellipticity is independent of κ_3. This result could also be obtained from (20) if we remove the χ_3 term that is anti-symmetric in reflection and multiply by the number of passes N. The conventional QED result is reproduced by substituting the value $\Delta\chi = -\dfrac{\mathbf{B}^2\alpha^2}{30\pi m_e^4}$:

$$\beta_{\mathrm{QED}} \approx -L\bar\omega \frac{\mathbf{B}^2\alpha^2}{60\pi m_e^4}\sin 2\theta. \tag{32}$$

4 Summary and conclusion

We have studied the dichroism and birefringence effects of laser light in a constant magnetic field from the viewpoint of effective field theory. As we assumed no new particles with a mass of order the laser frequency or below, the effective theory is for the electromagnetic field alone. We employ gauge and Lorentz symmetry to construct the quartic terms in the field tensor in the effective Lagrangian. Besides the standard structures occurring in QED, there is a new term, which violates parity. The coefficients of the terms are free parameters and are to be determined by a certain underlying theory.

The optical effects of the quartic terms are then explored for the linearly polarized laser in a transverse magnetic field. Due to parity violation by the new term, we distinguish experimental set-ups according to their properties under space reflection. For the asymmetric set-up, both parity-conserving and -violating interactions contribute to the ellipticity, but their maximum effects occur for different orientations of the laser polarization with respect to the magnetic field: the parity-conserving terms reach the largest effect when the polarization has an angle of $\pi/4$ to the field, while the parity-violating term has the largest contribution when the polarization is parallel or perpendicular to the field. Concerning the rotation of polarization, the parity-conserving terms never contribute to the order considered here, while the violating term results in a rotation of the same magnitude as the ellipticity in the ideal case, which, however, looks not practical for the laboratory experiment. For a symmetric set-up such as PVLAS, all parity-violating effects disappear; in particular, the ellipticity depends only on the difference of the coefficients parameterizing the parity-conserving interactions as happens in the conventional QED. If the PVLAS results are confirmed, it seems that we have to invoke some low-mass particles with nontrivial interactions with photons.

Note added. When this paper was being prepared for publication, a new preprint [31] appeared, in which the low energy polarization effects in a magnetic field of a hypothetical charged spin-s particle were studied. The particle was assumed to be in the representation $(0, s) + (s, 0)$ of the Lorentz group [32]. That approach corresponds in our notation to the case in which $\kappa_3 = 0$ and κ_1, κ_2 are given by the one-loop QED result of the particle. The conclusion reached in that approach is consistent with ours for this special case.

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